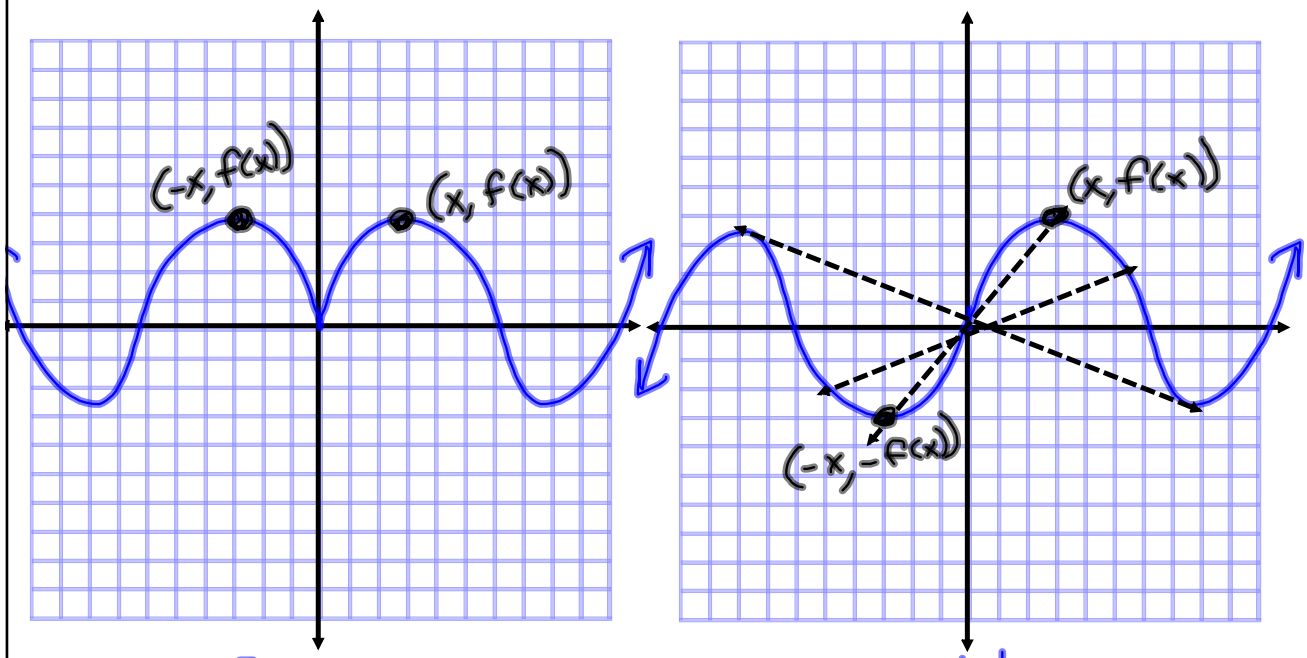
1.3 Properties of Functions

OBJECTIVES

1. Determine Even and Odd Functions from a Graph
2. Identify Even and Odd Functions from the Equation
3. Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant
4. Use a Graph to Locate Local Maxima and Local Minima
5. Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing
6. Find the Average Rate of Change of a Function

1.3	**Practice**	#s 1 – 20, 23, 25, 29, 31, 33, 35, 37, 39, 53, 55, 63
	Hand in	#s 15, 16, 19, 20, 21, 23, 25, 43, 59, 63
	Notes	**#1** The interval (2,5) is equal to the **set** $\{x \mid 2 < x < 5\}$. The textbook is happy with equating (2,5) with the **condition** $2 < x < 5$. I am not. **#s 17, 19, 29, 31** Write your answer in the form "Local max @ $x = 2$ of $y = 10$, giving the point (2,10) on the graph," to cement the concept. **#21a** The textbook and I differ on how to report intercepts. Here's one way to report them: (a) x - intercepts : (–2,0), (2,0); y - intercept : (0,3). **#s 33 – 34** Evaluate $f(-x)$. If it equals $f(x)$, f is even. It it equals $-f(x)$, it's odd. If it equals neither, then it's neither odd nor even.



Even
$f(-x) = f(x)$
Symmetric w.r.t. y-axis

Odd
$f(-x) = -f(x)$
Symmetric w.r.t. the origin.

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'Are You Prepared?'

$\{x \mid 2 < x < 5\}$

1. The interval $(2, 5)$ can be written as the inequality _____. (pp. A66–A67)
2. The slope of the line containing the points $(-2, 3)$ and $(3, 8)$ is _____. (p. 18)
3. Test the equation $y = 5x^2 - 1$ for symmetry with respect to the x-axis, the y-axis and the origin. (pp. 11–13)

No. Yes No

$f(x) = 5x^2 - 1$

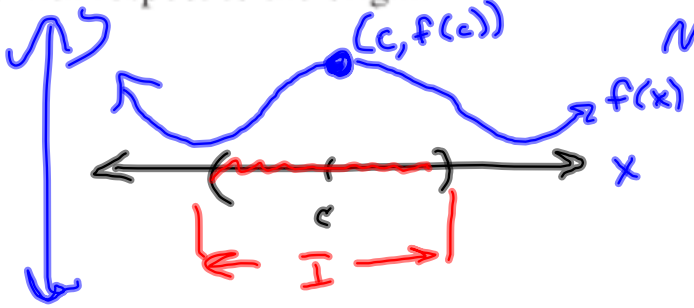
$f(-x) = 5(-x)^2 - 1$

$= 5x^2 - 1 = f(x)$ EVEN

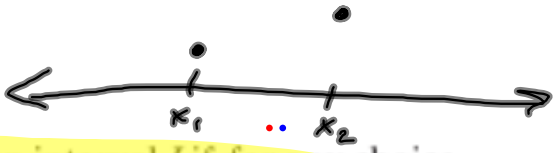
4. Write the point–slope form of the line with slope 5 containing the point $(3, -2)$. (p. 22)
5. The intercepts of the equation $y = x^2 - 9$ are _____. (pp. 10–11)

9. *True or False:* A function f has a local maximum at c if there is an open interval I containing c so that, for all x in I, $f(x) \le f(c)$. TRUE

10. *True or False:* Even functions have graphs that are symmetric with respect to the origin. FALSE



Concepts and Vocabulary

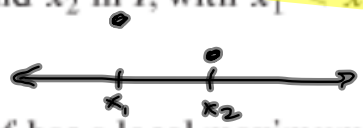


6. A function f is INCREASING on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

7. A(n) _____ function f is one for which $f(-x) = f(x)$ for every x in the domain of f; (an) ODD function f is one for which $f(-x) = -f(x)$ for every x in the domain of f.

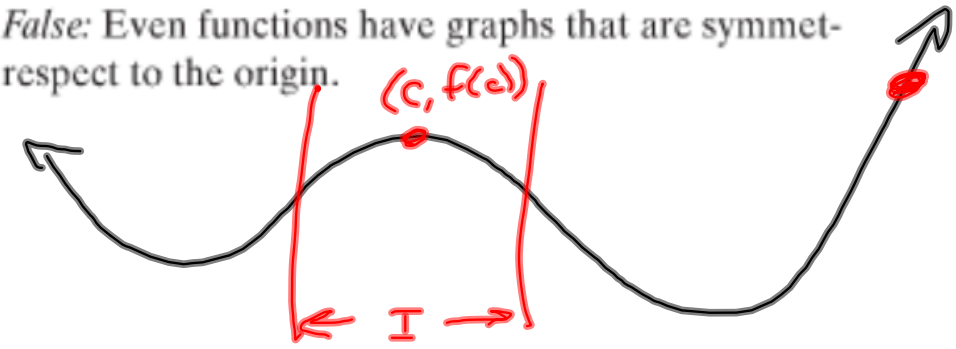
Even

8. *True or False:* A function f is decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.



9. *True or False:* A function f has a local maximum at c if there is an open interval I containing c so that, for all x in I, $f(x) \leq f(c)$.

10. *True or False:* Even functions have graphs that are symmetric with respect to the origin.

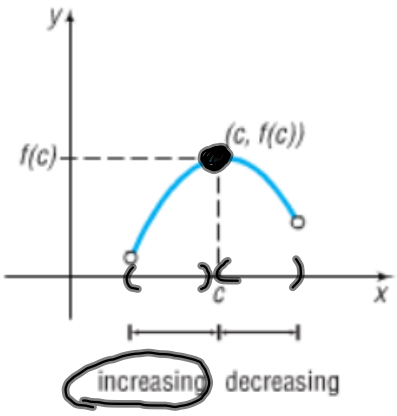


A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

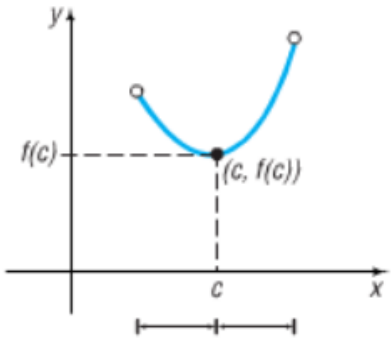
A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

A function f is **constant** on an open interval I if, for all choices of x in I, the values $f(x)$ are equal.

Notice that these definitions are *all* in terms of *open intervals*. The current Calculus textbook seems to be more of a *closed interval* thing, where you include the endpoints. *WE*, in this class, will speak only of *open intervals of increase/decrease/constancy.*



The local maximum is $f(c)$ and occurs at $x = c$.



The local minimum is $f(c)$ and occurs at $x = c$.

A function f has a **local maximum** at c if there is an open interval I containing c so that, for all x in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of f.**

A function f has a **local minimum** at c if there is an open interval I containing c so that, for all x in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of f.**

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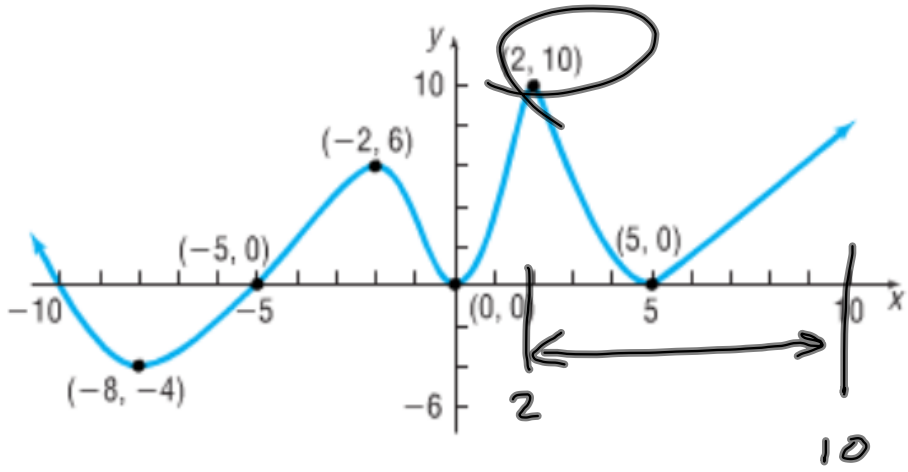


11. Is f increasing on the interval $(-8, -2)$? Yes

↳ x-values for which y-values are increasing.

13. Is f increasing on the interval $(2, 10)$? No

16. List the interval(s) on which f is decreasing. $(-\infty, -8) \cup (-2, 0) \cup (2, 5)$

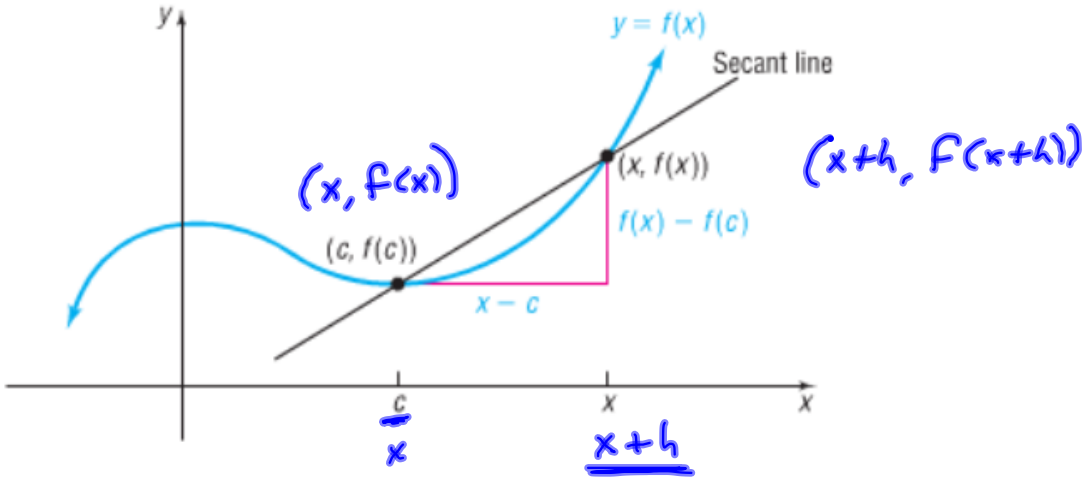
19. List the numbers at which f has a local maximum. What are these local maxima?

$x = 2, -2$, $y = 6, 10$, respectively.

The average rate of change of a function has an important geometric interpretation. Look at the graph of $y = f(x)$ in Figure 25. We have labeled two points on the graph: $(c, f(c))$ and $(x, f(x))$. The line containing these two points is called the **secant line**; its slope is

$$m_{\text{sec}} = \frac{f(x) - f(c)}{x - c} = \frac{f(x+h) - f(x)}{h}$$

Figure 25



Slope of the Secant Line

The average rate of change of a function equals the slope of the secant line containing two points on its graph.

If c is in the domain of a function $y = f(x)$, the **average rate of change of f** from c to x is defined as

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c} \qquad x \neq c \qquad \textbf{(1)}$$

In calculus, this expression is called the **difference quotient** of f at c.

Slope of the secant line, average rate of change, difference quotient - This is *all the same thing !!!*

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56. Find the average rate of change of $h(x) = x^2 - 2x + 3$

(a) From -1 to 1

(b) From 0 to 2

(c) From 2 to 5

Find the average rate of change of h from x to c, i.e., find the difference quotient for h.

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A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = f(x)$$

A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = -f(x)$$

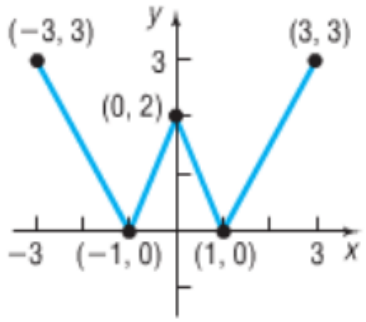
THEOREM:

A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

In Problems 21–28, the graph of a function is given. Use the graph to find:

(a) *The intercepts, if any*
(b) *The domain and range*
(c) *The intervals on which it is increasing, decreasing, or constant*
(d) *Whether it is even, odd, or neither*

22.



Problems 33–44, determine algebraically whether each function is even, odd, or neither.

35. $g(x) = -3x^2 - 5$

40. $f(x) = \sqrt[3]{2x^2 + 1}$

44. $F(x) = \dfrac{2x}{|x|}$

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64. Constructing an Open Box An open box with a square base is required to have a volume of 10 cubic feet.

(a) Express the amount A of material used to make such a box as a function of the length x of a side of the square base.

(b) How much material is required for a base 1 foot by 1 foot?

(c) How much material is required for a base 2 feet by 2 feet?

(d) Graph $A = A(x)$. For what value of x is A smallest?

Similar problem situation to #63, which I'm assigning for homework. But for #64, they're giving you slightly different info and are looking for slightly different things...

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